Self-force as probe of internal structure

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The self-force is the result of a nonlocal interaction between the field created by an electric charge and the spacetime curvature. In principle the self-force depends on all aspects of the spacetime, both local and remote.

In the case of a static charge outside a spherical distribution of matter, the self-force depends on the body’s internal structure. What does the self-force tell us about this internal structure?
Static charges in static, spherically-symmetric spacetimes

Key works:

- Smith and Will (1980): Schwarzschild black hole
- Unruh (1976): Inside a thin shell
- Burko, Liu, and Soen (2001): Inside and outside a thin shell
- Drivas and Gralla (2011): Outside a matter distribution

Drivas and Gralla showed that

\[ F^r = \frac{e^2 M}{r^3} + O(r^{-5}) \]

for any matter distribution; the self-force is universal to leading order in \( r^{-1} \).

They showed also that the dependence on internal structure occurs at order \( r^{-5} \), but did not explore this dependence.
This work

To explore the dependence on internal structure, we compute the self-force acting on a static electric charge outside a static, spherically-symmetric, relativistic polytrope.

The rest-mass density $\rho$, pressure $p$, and thermodynamic energy density $\epsilon$ satisfy the equations of state

$$p = K\rho^{1+1/n}, \quad \epsilon = np$$

with $K$ a scaling constant and $n$ the polytropic index.

For given $K$ and $n$, polytropic stellar models form a family parameterized by the central density $\rho_c$.

They have a total mass $M$ and a radius $R$. 
Following Drivas and Gralla, we compute the self-force difference

\[ \Delta F^r = F^r_{\text{polytrope}} - F^r_{\text{black hole}} \]

where

\[ F^r_{\text{black hole}} = \frac{e^2 M}{r^3} (1 - 2M/r)^{1/2} \]

with an exponentially-converging mode-sum method. We analyze the dependence of \( \Delta F^r \) on the equation of state.
The self-force difference is plotted as a function of $r/M$; all polytropes have $R/M = 15$. 