Strong, Time-Dependent Electromagnetic Fields in the Presence of Strong, Time-Dependent Gravity

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Outline: Relativistic MHD of Black Hole Jets, Accretion, and Formation

- Four topics, from the outer jet lobes to the hole formation
  - M87 knots as MHD shocks in a Poynting-dominated jet (Nakamura, Garofalo, & Meier; 10 min)
  - Simulation of the hard accretion state as a radiatively-cooled Magnetically-Dominated Accretion Flow (Fragile & Meier; 5 min)
  - Numerical Constrained Transport as a Discrete Differential Geometry technique for evolving everything: EM & GR fields plus charge and matter sources (DLM; 10 min)
THE OUTER JET AND LOBES OF M87: WHAT THEY TELL US ABOUT JET DYNAMICS

Nakamura, Garofalo, Meier
The Fanaroff & Riley Classification and Correlation

FR I (M 84 / 3C 272.1)

“1” emission region near galaxy

FR II (3C 47)

2 emission regions away from galaxy

- The Fanaroff & Riley correlation:
  - FR Class I sources are low luminosity,
  - FR Class II are high luminosity, with the break at $P_{178} \sim 10^{25.3}$ W/Hz/Sr

- The FR I / FR II break is a strong function of galaxy OPTICAL luminosity ($\sim L_{\text{opt}}^2$) (Owen & Ledlow, AJ, 112, 9-22, 1996)
Cygnus A and the Blandford-Rees Hydrodynamic Model for Lobes & Hot Spots

**Blandford & Rees’ 1974** hydrodynamic model for the hot spots and lobes has withstood the test of time:

With only the HR74 map of Cyg A to go on, they deduced that FR II sources
- were powered by jets
- produced a strong reverse “Mach disk” (hot spot) & forward “bow” shocks
- produced a hot cocoon of post-shock jet material that surrounded the jet
- **FR II sources, therefore look like hydrodynamic (HD) jets**
Simulations of **MHD** Jets

- 1\textsuperscript{st} 2-D simulations of magnetized jets performed in 1980s:
  - Lind, Payne, Meier, Blandford (1989)
  - Clarke, Norman, Burns (1986)

- Results
  - Jets with high $\beta_p = p_{\text{gas}}/p_{\text{mag}} \approx \frac{2}{3} (B_{\text{eq}}/B)^2 >> 1.0$ (HD jets) look like an FR II
  - Jets with low $\beta$ ($< 2.0$) develop fast, leading “nose cones”, forced forward by a strong toroidal magnetic field
  - Nose cone contains several slow shock pairs
  - In general, an FR II radio source does NOT have the morphology of a magnetized jet with an equipartition toroidal magnetic field
  - How about FR I sources?
MHD Simulations of Jets (cont.)

- 3-D simulations of magnetized jets:

- Results
  - Even 3-D simulations show a shock system plus nose cone-like structure
  - "Nose cones" are facilitated by
    - Poynting flux domination (1; LPMB; Komissarov)
    - Steeper external pressure gradient (B)
  - At late times the slow shock pair develops a kink instability between the slow-mode shocks
Consequences of FR I Lobe Morphology: The Case of M87

- If FR II jets are supersonic hydrodynamic jets, then what are FR I sources?
  - Model #1: Transonic FR II flows that spontaneously decelerate, inflate, decelerate, … (Bicknell 1985, 1995)
  - Model #2: Modest Mach number flows that decelerate and inflate by interacting with an external shock (Norman, Burns, Sulkanen 1988)
  - Model #3: Magnetically dominated jets that never became fully kinetic (Nakamura, Garofalo, Meier 2009)

--- an extraordinary, detailed, paradigm-shifting, model

Motivation for MHD model:

- Two knots have very strong measured magnetic fields:
  - Knot HST-1 (Perlman et al. 2003; 10 mG)
  - Knot A (Stawarz et al. 2005; 100 µG < B < 1 mG)
- Inter-knot jet particle pressure << ambient external pressure (Sparks et al. 1996)
- A ⇔ C helical kink
  ⇒ strong magnetic forces in jet
The M87 Jet as a Poynting-Dominated Flow

1-D Super-Fast, 4-Shock MHD M87 Jet Simulation

Owen et al. (1989)
Biretta & Meisenheimer (1993)
Sometime between 2005 December and 2006 February, the knot HST-1c split into two approximately equally bright features: a faster moving component (c1; 4.3c ± 0.7c) and a slower moving trailing feature (c2; 0.47c ± 0.39c).

HST-1 is essentially stationary (< 0.25c), and it appears to be the source of successive components, each of which splits into forward/reverse bright knots downstream

Our proposed model:

The superluminal components in M87 jet are
- relativistically propagating internal MHD shock fronts (not "blobs")
- ejected from HST-1, not from the core itself!

(Nakamura, Garofalo, & Meier 2009)

Working on complete M87 model: more papers to come …
LAUNCHING JETS FROM THE HARD STATE ACCRETION DISK: HOW DOES A LOW-LUMINOSITY ACCRETION FLOW SET UP A STRONG POLOIDAL MAGNETIC FIELD AND LAUNCH A JET?

Fragile & Meier
Very Important Synthesis of Jet-Disk Connection (BH Accretion States DO MATTER)

- A new and very important color-magnitude plot: the FBG diagram (Fender, Belloni, Gallo 2004) for jet-producing binary X-ray sources
  - Like the HR diagram, but in X-rays, and color axis is reversed
  - HIGH and LOW refer to 2-10 keV X-ray flux
  - High/Soft state at upper left
  - Low/Hard state at lower right
  - Jet states are at top and right
  - Explosive jets occur only on transition from Hard state to Soft state

- A tremendous amount has been learned recently about how actual observed accreting black hole systems behave when they are producing jets
  - Black holes follow a prescribed path on the X-ray color (soft vs. hard) – magnitude (low vs. high intensity) diagram: takes days/hours, not Myr
  - Inner radius of cool disk decreases as spectrum becomes softer (the “truncated disk model”)
  - Jet velocity increases as disk spectrum becomes softer

- Implication: In hard state, slow ($\Gamma < 2$) jet is NOT launched from vicinity of black hole, but from 10 -100 $r_g$ instead
The three stages of MDAF formation, predicted from analytic models (Meier 2004)

- Regular non-radiative (ADAF) flow forms for $r > \sim 100 \, r_g$.
  Relativistic electron synchrotron and Compton cooling are important when $T_e > \sim 10^{9.7} \, K$ ($r < \sim 100 \, r_g$)

- Cooling will reduce thermal pressure and therefore reduce plasma $\beta \equiv p_{\text{gas}} / p_{\text{mag}} \Rightarrow < 1$
  (magnetically dominated)

- Magnetic domination will turn off magneto-rotational instability that drives MHD turbulence, creating an inward-facing corona.

- Open field lines will create conditions conducive to driving jets, but from outer edge of MDAF and maybe rotational QPOs.

- Final structure should look similar to "black hole magnetosphere" models of Tomimatsu & Takahashi (2001) and Uzdensky (2004)

$V_{\text{jet}} \sim V_{\text{esc}} \approx 0.3 \, c$

Tomimatsu & Takahashi (2001); Uzdensky (2004)
Studies of Cooled Black Hole Accretion Flows and Possible Development of Jet-Producing Magnetospheres

(continued)

• We are using Chris’ COSMOS++ code (Anninos, Fragile, & Salmonson 2005) to test out each stage of this model

• We added Esin et al. (1996) cooling functions (Bremsstrahlung, Synchrotron, Comptonization) to COSMOS++

• Results
  – Cooling Does, indeed, produce a strong-field accretion flow ($\beta \rightarrow 1$) as flow approached black hole
  – Results quantitatively agreed with analytic predictions (“transition region” solution)
  – Choice of parameters did not allow strong MDAF, so now tooling up for $\beta < 1$ simulations

• Questions for new simulations
  – Does a true black hole magnetosphere form in some circumstances? If so, when and what controls its formation?
  – Do the resulting rotating black hole magnetospheres say anything about jet launching from hard state objects?
CONSTRAINED TRANSPORT: 
A DISCRETE DIFFERENTIAL
GEOMETRY ON A 4-DIMENSIONAL
MANIFOLD

TOWARD
A FULLY INTEGRATED METHOD
FOR SIMULTANEIOUSLY EVOLVING
GR AND E&M FIELD PROBLEMS
ALONG WITH THEIR CONSTITUENT
CONSERVATION LAWS

Meier (& Miller)
The Ultimate Goal: Simulate EM Gravitational Collapse

To solve the problem of Electromagnetic Gravitational Collapse, we need to evolve both the gravitational and electromagnetic fields and their sources (matter and charge).

![Images of Neutron Star Binary Coalescence and Black Hole Binary Coalescence](image)

<table>
<thead>
<tr>
<th>Physics</th>
<th>Non-Relativistic Equations</th>
<th>Relativistic Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity Field</td>
<td>( \nabla^2 \psi = 4\pi G \rho ) \hspace{1cm} [\psi = GM/r ]</td>
<td>( \mathbf{G} = 8\pi G \ T/c^4 )</td>
</tr>
<tr>
<td>Matter</td>
<td>( \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 ) \hspace{1cm} ( \frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = -\nabla p + \mathbf{J} \times \mathbf{B}/c - \rho \nabla \psi ) \hspace{1cm} ( \frac{\partial (\rho \mathbf{e})}{\partial t} + \nabla \cdot (\rho \mathbf{e} \mathbf{V}) = - (\rho + \mathbf{e}) \nabla \cdot \mathbf{V} )</td>
<td>( \nabla \cdot (\rho \mathbf{U}) = 0 ) \hspace{1cm} ( \nabla \cdot \mathbf{T} = 0 )</td>
</tr>
<tr>
<td>EM Field</td>
<td>( \frac{\partial \mathbf{B}}{\partial t} + c \nabla \times \mathbf{E} = 0 ) \hspace{1cm} ( \nabla \cdot \mathbf{B} = 0 ) \hspace{1cm} ( \frac{\partial \mathbf{E}}{\partial t} - c (\nabla \times \mathbf{B}) = -4\pi \mathbf{J} ) \hspace{1cm} ( \nabla \cdot \mathbf{E} = 4\pi \rho_q )</td>
<td>( \nabla \cdot *\mathbf{F} = 0 ) \hspace{1cm} ( \nabla \cdot \mathbf{F} = 4\pi \mathbf{J}/c )</td>
</tr>
<tr>
<td>Charge/Current</td>
<td>( \frac{\partial \rho_q}{\partial t} + c \nabla \cdot \mathbf{J} = 0 ) \hspace{1cm} ( \mathbf{E} = -\nabla \times \mathbf{B}/c ) (Ohm’s law ( \sigma \equiv 1/\eta \rightarrow \infty ))</td>
<td>( \nabla \cdot \mathbf{J} = 0 ) \hspace{1cm} ( \mathbf{U} \cdot \mathbf{F} = ?? )</td>
</tr>
</tbody>
</table>
Constrained Transport for MHD  
(Evans & Hawley 1988)

- MHD constrained transport: evolving $\dot{B} = -c \nabla \times E$ automatically maintains the constraint $\nabla \cdot B = 0$.

- To do this, one staggers the grid in space and time:

  - At $t = t^0$, $B = \nabla \times A$ and
    \[
    \nabla \cdot B = B_x^+ - B_x^- + B_y^+ - B_y^- + B_z^+ - B_z^- \\
    = (A_{z4} - A_{z2}) - (A_{y4} - A_{y2}) - (A_{z3} - A_{z1}) + (A_{y3} - A_{y1}) \\
    + (A_{x4} - A_{x2}) - (A_{z4} - A_{x3}) - (A_{x3} - A_{x1}) + A_{x2} - A_{x1} \\
    + (A_{y4} - A_{y3}) - (A_{x4} - A_{x3}) - (A_{y2} - A_{y1}) + (A_{x2} - A_{x1}) = \nabla \cdot \nabla \times A = O(\varepsilon_r)
    \]

  - Then at $t = t^{1/2}$, similarly, $\nabla \cdot \nabla \times E = O(\varepsilon_r)$, so $\nabla \cdot \dot{B} = O(\varepsilon_r)$

  - So, at $t = t^1$,
    \[
    \nabla \cdot B^1 = \nabla \cdot B^0 + \Delta t \left( \nabla \cdot \dot{B}^{1/2} \right) = O(\varepsilon_r)
    \]

- So, if we put $A$ on cube edges at $t=t^0$ and $E$ on cube edges at $t=t^{1/2}$, we can evolve $B$ forward in time and keep $\nabla \cdot B = O(\varepsilon_r)$ without any additional effort.
CT for Electrodynamics (Yee 1966)

- Actually, a more complete version of CT was invented by an **engineer**: the FDTD (finite-difference time-domain) algorithm used in antenna design and analysis.
- In full electrodynamics, **both** of Maxwell’s equations and **both** of the constraints must be propagated:
  \[
  \begin{align*}
  B &= -c \nabla \times E \\
  E &= c \nabla \times B - 4\pi J \\
  \nabla \cdot B &= 0 \\
  \nabla \cdot E &= 4\pi \rho_q
  \end{align*}
  \]
  creating the need for **three interlaced updates** (one each for $E$, $B$, and $\rho_q$):

  ![Diagram showing interlaced updates](image)

- In 4-D form, the Yee algorithm looks much simpler:

  ![Diagram showing 4-D form](image)

- Staggering the grid implicitly satisfies the **Bianchi identities** ($\nabla \times \nabla \phi = 0$; $\nabla \cdot \nabla \times A = 0$) to machine accuracy, and this implicitly transports the constraints.
- Furthermore: the law of conservation of charge ($\dot{\rho}_q = -\nabla \cdot J$) must be solved in a staggered grid manner in order to properly transport the inhomogeneous constraint and solve the E & M field.

- The electrodynamics CT problem suggests a natural, simple, and elegant method for staggering finite difference grids in 4-D
- Special cases have interesting forms
  - Kronecker delta $\delta^{\alpha}_\lambda$: $4 \times 1$s at cell corners; $12 \times 0$s at cube faces
  - Other identity tensors $\delta_{\alpha\beta\gamma\lambda\mu}$: $\pm 1$ at corners; $0$ otherwise
  - Levi-Civita tensor $\varepsilon_{\alpha\beta\gamma\delta}$: $\pm \sqrt{-g}$ at hypercube body centers; $0$ otherwise
- Gives rise to the concept of a dual mesh
  - Shift origin to hypercube-centered point to create the dual mesh
  - $\varepsilon_{\alpha\beta} IS \delta_{\alpha\beta}^\lambda$ as viewed from the dual mesh ($\delta_{\alpha\beta}^\lambda$)
  - As viewed from the dual mesh the Maxwell tensor, the dual of $F$ ($M = *F$), is simply $F \sqrt{-g}$
- To paraphrase J. Wheeler, “A staggered grid has deep geometric significance”
Notes on Bianchi Identities in CT for EM & GR

- **Centrally-differenced** CT is
  - Exact for E & M — to machine accuracy!
    - \( (\sqrt{-g} (\sqrt{-g} F^{\alpha\beta})_{\beta} )_{,\alpha} = O(\epsilon_r) \)
  - Exact for GR — in Riemann-normal coordinates only (\( \Gamma = 0 \))
    - \( R_{\alpha\beta;\gamma\delta,\epsilon} = O(\epsilon_r) \)
  - “Almost” exact for GR — in *global* coordinates
    - \( R_{\alpha\beta;\gamma\delta,\epsilon} \sim \partial^2 \Gamma + \Gamma \partial \Gamma + \Gamma \Gamma \Gamma \)
    - \( \Gamma \partial \Gamma \) terms *also* commute to machine accuracy!
    - \( \Gamma \Gamma \Gamma \Gamma \) terms DO NOT commute, BUT THEY NEARLY DO SO
      with adaptive gridding and high-order differencing in regions of large \( \Gamma \)

- In order to properly transport constraints without losses (to near machine accuracy), we need ALL OF THE FOLLOWING
  - \( G^{\alpha\beta} = 8\pi T^{\alpha\beta} \)
  - \( T^{\alpha\beta} = T^{\beta\alpha} \) (T symmetry, since grid staggering ensures \( G^{\alpha\beta} = G^{\beta\alpha} \))
  - \( G^{\alpha\beta} ;_\beta = 8\pi T^{\alpha\beta} ;_\beta \)

- That is, we need the natural symmetries in all the tensors AND we need to apply the SAME DIVERGENCE OPERATOR to matter and GR fields alike
Practicalities: Does CT Work for the GR Gauge Field?

- **Tests with no sources** (Miller & Meier 2005, unpub):
  - Diagonal test (metric Gowdy cosmology):
    - CT is stable and convergent, AND nearly equivalent to best finely-tuned methods (BSSN)
  - Off-diagonal test in $Z$ (gauge plane wave):
    - CT is stable and convergent
  - Off-diagonal test in $XY$ (Bondi plane wave):
    - CT is stable and convergent but only if Christoffel symbols are evolved as a set in a strongly hyperbolic manner along with the metric!
  - CT by itself, therefore, does not guarantee stability

**Fully Hyperbolic CT:**
Evolve Christoffels; Evolve Metric using Christoffels

**Partially Hyperbolic CT:**
Evolve Metric; Compute Christoffels Using Centered Spatial Derivatives of Metric
CT with Field Sources

- Maxwell Field equations
  \[ \nabla \cdot F = 4\pi J \]
- Field sources
  \[ J = \text{charge-current 4-vector} \]
- Field Bianchi identities
  \[ \nabla \cdot (\nabla \cdot F) = 0 \]
- Implied conservation law
  \[ \nabla \cdot J = 0 \]

The conservation of charge is a direct result of E & M

- Einstein Field equations
  \[ G = 8\pi T \]
- Field sources
  \[ T = \text{stress-energy-momentum tensor} \]
- Field Bianchi identities
  \[ \nabla \cdot G = 0 \]
- Implied conservation law
  \[ \nabla \cdot T = 0 \]

The conservation of momentum and energy is a direct result of GR

And, just as there is a 4-D staggered-grid technique for integrating the conservation of current, there also should be a 4-D staggered-grid technique for integrating the conservation of energy and momentum.
Staggered-Grid Algorithms for Fluid Dynamics

- Consider $\nabla \cdot T = 0$ in flat space
- $T = [\rho + (p + \varepsilon)/c^2] \mathbf{U} \mathbf{U}^T + p \eta$

\[
\begin{bmatrix}
T_{tt} & T_{tx} & T_{ty} & T_{tz} \\
T_{tx} & T_{xx} & T_{xy} & T_{xz} \\
T_{ty} & T_{xy} & T_{yy} & T_{yz} \\
T_{tz} & T_{xz} & T_{yz} & T_{zz}
\end{bmatrix}
\]

- Note that the stress-energy-momentum tensor is symmetric
- That is, the energy flux $T_{xt}$ equals the momentum conserved variable $T_{tx}$
- So, once the $T_{ij}$ are computed, the $T_{ij}$ and $T_{tt}$ updates should follow immediately with no additional effort
Does CT Work for Fluid Dynamics?

- **Tests with no fields** (Meier 2009, unpub): Simple shock tube
  - Lax-Wendroff test (**fully centered** differencing):
    - CT works, but contains oscillations both at shock and contact discontinuity
  - Lax-Wendroff plus artificial bulk viscosity:
    - CT works, but oscillations still persist at contact discontinuity
  - Nakamura 2-step hyperbolic scheme (LW+Godonov):
    - Works, but centered differencing must be discarded FOR ALL EQUATIONS.
    - Just like the field evolution tests, strongly-hyperbolic evolution algorithms must be used for all sets of evolution equations
Bottom Line: Hyperbolic CT Needed

• As a discrete differential geometry, CT has tremendous power and capability

• Central-differenced CT works fine for evolving the sourceless electromagnetic field

• However, central-differenced CT is unstable in its evolution of the Einstein field and produces undesirable results in the evolution of its conservation laws

• To proceed further we need to completely recast CT with hyperbolic, not central, differential operators

• Mark Miller’s 8th-order difference operators look very interesting