Granular Mechanics and Dusty Plasmas
Christine Hartzell, Daniel Scheeres
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Granular Flows Summer School

University of Colorado Boulder
Motivation: Dust

Figure Credit: Colwell et al. *J. Aerospace Engineering* 2009.

Figure Credit: Berg et al. *Interplanetary Dust and Zodiacal Light* 1976.

Figure Credit: Robinson et al. *Nature* 2001.
Intro to Electrostatic Levitation

Neutral Plasma

Plasma Sheath

Photoemission

Equilibrium Current
Surface Potential

Solar Wind and Photoelectron Reimpactor
Outline

• Dust Particle Launching

• Experimental Problems

• Electrostatic Levitation
Equation of Motion

- Model of seismic shaking:
  \[ a = A_t \sin(\Omega_t t)\hat{x} + A_n \sin(\Omega_n t)\hat{y} \]
  \[ a_t = A_t \sin(\Omega_t t) \]
  \[ a_n = A_n \sin(\Omega_n t) \]

- Equation of Motion (vertical shaking):
  \[ m_d \ddot{y}_{rel} = F_{es} + F_{grav} + F_{co} + N - m_d a_n \]

- Conditions:
  - If \( \ddot{y}_{rel} \geq 0 \), then the particle is said to be \textit{separated} from the surface. The cohesive force disappears in subsequent motion.
  - If \( \ddot{y}_i > 0 \), the particle is said to be \textit{launched}. The cohesive force disappears in subsequent motion.
  - EOM is valid only until particle is no longer in contact with surface (gravity is assumed to be constant).
Force Models

- **Gravity:**  \[ F_{grav} = -\frac{4}{3} \pi r_d^3 \rho g_s \]

- **Electrostatic force:**  \[ F_{es, gen} = QE \]
  
  - Assume:  \[ Q = EA\epsilon_0 \]

  \[ F_{es} = E^2 \pi r_d^2 \epsilon_0 \]
Force Models (Cohesion)

- Cohesion (Perko[23]):

\[
F_{co} = \frac{B}{48(t + d)^2} \frac{r_1 r_2}{r_1 + r_2}
\]

- Assume:

\[
S = 1.32 \times 10^{-10} / t
\]

\[
B = 4.3 \times 10^{-20} \text{ J}
\]

\[
F_{co} = -5.14 \times 10^{-2} S^2 r_d
\]
Electric Field Required

• What electric field strength is required to separate a particle from a surface (\( \ddot{y}_{rel} > 0 \))?

\[
F_{es} \geq m_d a_n - F_{grav} - F_{co}
\]

\[
E_{req} \geq \left[ \frac{4}{3 \epsilon_0} r_d \rho (g_s + a_n) + \frac{5.14 \times 10^{-2} S^2}{\pi \epsilon_0 r_d} \right]^{1/2}
\]

• If \( a_n = 0 \), then the electric field is that required to launch a particle.

<table>
<thead>
<tr>
<th></th>
<th>Moon</th>
<th>Eros</th>
<th>Itokawa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity</td>
<td>1.622 m/s²</td>
<td>0.0055 m/s²</td>
<td>8.603x10⁻⁵ m/s²</td>
</tr>
<tr>
<td>Radius</td>
<td>1737.1 x10³ m</td>
<td>8420 m</td>
<td>165 m</td>
</tr>
</tbody>
</table>
- \( E_{\text{req}} \gg E_{\text{criswell}} \)
- Curve minima at large particle sizes (even at S=0.1)
- Curve minima shifted right for asteroids
- See also: Hartzell and Scheeres. *The role of cohesive forces in particle launching on the Moon and asteroids.* Planetary and Space Science.
- Lunar gravity only
- Seismic accelerations do not significantly impact $E_{\text{req}}$ for launching
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Experimental Dust Lofting

• Two main dust lofting experiments:
  – Xu Wang et al. 2009, spreading dust mound
  – Flanagan and Goree 2006, dust release from spinning sphere

• With Wang’s values
  $F_{es} < F_{grav}$ !!!

Figure Credit: Wang et al. JGR 2009.
Charge Amplification

• We calculate the amount charging level beyond that predicted by Gauss’ law that is required to explain experimental results.

• $4.04 \times 10^4 < C_{\text{amp}} < 1.04 \times 10^9$

• Use $C_{\text{amp}} = 5.01 \times 10^6$

• Can this level of charge amplification occur in reality?
• Lofting possible in terminator
• Still not possible elsewhere (subsolarm=10^{-1} V/cm)
• Easiest size to loft does not change
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What is Particle Levitation?

- Lacking a feasible launching mechanism, arbitrary initial conditions are chosen.
Plasma Physics Disclaimer

Nitter et al. Model
- Predicts two sheath types
- Non-monotonic sheath has been suggested as more energetically favorable.
- Presence of non-monotonic sheaths supported by PIC models and Lunar Prospector data.

Colwell et al. Model
- Simpler, analytically-described sheath.
- Only photoelectron density varies with altitude.
- Has been used in previous dust motion studies by Colwell.

Non-Monotonic Sheath
Description of Sheath

\[ \dot{q} = \sum I(h, q) \]

\[ m_a \ddot{h} = qE - \frac{m_d g_s}{\left( \frac{h}{r_c} + 1 \right)^2} \]
Linear Stability Analysis

• Define: $F(q, h) = \dot{q}$  $G(q, h) = \ddot{h}$

\[
\begin{bmatrix}
\delta \dot{q} \\
\delta \dot{h} \\
\delta \ddot{h}
\end{bmatrix}
= A
\begin{bmatrix}
\delta q \\
\delta h \\
\delta \dot{h}
\end{bmatrix}
A = \begin{bmatrix}
\frac{\partial F}{\partial q} & \frac{\partial F}{\partial h} & 0 \\
0 & 0 & 1 \\
\frac{\partial G}{\partial q} & \frac{\partial G}{\partial h} & 0
\end{bmatrix}
\]

• From A, compute linearized stability by solving for eigenvalues ($x$):

\[
x^3 - F_q x^2 - G_h x + F_q G_h - F_h G_q = 0
\]

• Eigenvalues give approximate oscillation frequency and decay constant

• Eigenvectors can be used in nonlinear stability analysis
Equilibria

Black: [0,1) electrons  
Red: [1, 10) electrons  
Blue: [10, 100) electrons  
Green: >100 electrons
Timescales of Motion

Rotation Periods:
- Itokawa: 12hrs
- Eros: 5 hrs
- Moon: 27 days
State Space

~3micron particle above Itokawa
State Space

Altitude (m)

Altitude Rate (m/s)
Fate Plots

Initial Altitude: 1.57 m

Initial Charge: 211 e
Conclusions

• Earlier work did not include cohesion when researching electrostatic dust lofting
  – Makes a big difference!
  – Experimental work is also lacking

• Have an increased understanding of levitation after doing dynamical systems analysis
  – Informs which particles will levitate in situ
  – Predictions of where to observe levitation
Future Work

- Complete state space exploration for electrostatic levitation
- Model dust motion about 3D asteroid (with accurate gravity)
- Experimental work on cohesion in plasma environment
Questions?

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