2011 Interdisciplinary Summer School: Granular Flows

IMPACTS

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Leader of the group of Planetology
Asteroid Mathilde (50 km)

- 1.3 g/cm³
- C-type
- low albedo
- (<0.1)

Asteroid Eros (23 km)

- 2.7 g/cm³
- S-type
- high albedo
- (>0.15)

Asteroid Itokawa (350 m)

- 1.9 g/cm³
- S-type

Note: even two bodies of same spectral type can be very different!

Great diversity of structures

⇒ bulk density:

smaller for lower albedo objects

Presence of regolith on all these bodies
Rock fragmentation:

A Modeling Challenge
Numerical methods

- Eulerian Hydrocodes based on grid-method
- Lagrangian Hydrocodes based on the 3D Smooth Particle Hydrodynamic (SPH) method:
  - To simulate non-porous solids, standard SPH was extended to include a strength and fracture model (Benz & Asphaug 1994)
  - Recently, porosity models were included based on different relations between state variables (Jutzi et al. 2008, Wunneman et al. 2006, Speith et al. ??).
Numerical Simulations of the fragmentation phase

- Solve conservation equations (using your favorite numerical method)
  - mass conservation
  - momentum conservation
  - energy conservation

- Define material properties
  - equation of state
  - elasticity/plasticity model
  - damage model
  - **NEW:** model of microporosity
  - ...

- Testing and testing
  - analytical solutions
  - laboratory experiments
  - code comparisons
  - observations/measurements in situ
  - ...

- laboratory experiments
  + characteristics of small bodies
Material Behavior: Three regimes

EOS

Single species EOS
\( e(P, \rho) \)

Single species EOS
\( e(P, \rho) \)

Mixture Theory
(including porosity)

Solids

Stress-Strain Equations

Flow, fracture, failure

Yield

Flow & Failure

Fracture

Yield surface,
Flow rule

Fracture Criteria

\( P>>\rho c^2 \)

\( P\sim\rho c^2 \)

\( P<<\rho c^2 \)
Equations

1) momentum conservation

\[ \frac{d \nu_i}{d t} = \frac{1}{\rho} \frac{\partial \sigma_{ij}}{\partial x_i} + \frac{\partial \phi}{\partial x_i} \]

stress tensor  self-gravity

\[ \sigma_{ij} = -P \delta_{ij} + S_{ij} \]

pressure  deviatoric stresses

2) mass conservation

\[ \frac{d \rho}{d t} = -\rho \frac{\partial \nu_i}{\partial x_i} \]

3) energy conservation

\[ \frac{du}{dt} = -\frac{P}{\rho} \frac{\partial \nu_i}{\partial x_i} + \frac{1}{\rho} S_{ij} \dot{\varepsilon}_{ij} \]

PdV term  elastic energy

\[ \dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial \nu_i}{\partial x_j} + \frac{\partial \nu_j}{\partial x_i} \right) \]
4) elasticity: Hooke's law

\[ \Delta \frac{L}{l} = \epsilon = \frac{\sigma}{E} \]

\( E \): Young's modulus

\[ \frac{dS_{ij}}{dt} = 2\mu \left( \dot{\epsilon}_{ij} - \frac{1}{3} \delta_{ij} \dot{\epsilon}_{kk} \right) + S_{ik} R_{jk} + S_{jk} R_{ik} \]

deformation terms \hspace{10cm} \text{rotation terms}

with the rotation rate tensor:

\[ R_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i} \right) \]
5) stress limiters

- von Mises (plasticity)

\[ \sqrt{3} \bar{\sigma} - Y_0 = 0 \]
\[ Y_0: \text{Yield strength} \]
\[ \bar{\sigma} = \sqrt{\frac{S_{ij} S_{ij}}{2}} \]
\[ \text{equivalent stress} \]

6) equation of state:

\[ P = f(\rho, u, \alpha, x, ...) \]
with \[ \alpha: \text{porosity} \]
\[ x: \text{chemical composition} \]

- multi-material
- multi-phase description
Strength:

The Mohr-Coulomb (or Drucker-Prager) model:

- Shear strength
- Cohesion
- Rocks
- “Angle of Friction”
- Sand
- Water
- Pressure
- Tensile region
- Compressive region
Yield depends on pressure

Cohesion

Tensile strength

Some real data

Sandstone data (after Hoek & Brown, 1980)

$\sigma_c/\sigma_o$

ANGLE OF FRICTION

$\sigma_3/\sigma_c$
Damage

feed back on dynamics: Damage $D$

$$D(t) = \frac{V(t)}{V} = \frac{4}{3} \pi \left[ c_t (t-t_0) \right]^3$$

$t_0$: activation time

0 ≤ $D$ ≤ 1

undamaged full shear and strength
totally damaged no shear and no strength
Strength

A rock has each of:

- Tensile strength
- Shear strength (cohesion) ~same as tensile
- Compressive strength ~5-7* tensile
The “F” words:
Flow, Fracture and Failure

Models for these fall into three groups:

• “Degraded Stiffness”, no explicit flow or fracture.

• “Flow” including plasticity and damage, used to model microscopic voids and cracks leading to an inability to resist stress.

• “Fracture”, involving actual macroscopic cracks and voids which are tracked, leading to an inability to resist stress.
The Grady-Kipp Model

- It is a **Tensile Brittle Fracture Mechanism**
  - For fragmentation in mining
- One-Dimensional Model
- Synthesized for constant strain rate histories only
- Governed by Crack Distributions (Weibull) and growth
- Implies rate and size-dependent strength

But Attractive Physics
There exists an initial distribution of incipient flaws in the target

Weibull distribution:

\[ N(\varepsilon) = k \varepsilon^m \]

where:
N = density number of flaws activating at or below the strain \( \varepsilon \)
k, m: Weibull parameters (large m = more homogeneous material)

\[ \varepsilon_{\text{min}} = (1/ kV)^{-m} \]

Larger targets (volume V) activate largest crack at lower strain

⇒ Larger targets are weaker
Tensile fracture depends strongly on strain rate

Strength v. Strain Rate from Various Studies

- Concrete (plain)
- Concrete (polyester)
- Limestone (Oakhall)
- Oil Shale (80ml/kg)
- Arkansas Novaculite
- Westerly Granite (Lipkin)
- H&H Granite (Crack Distribution)
- Fully Cracked, Large (Various Materials)
- Melosh et al. (Basalt)
- Dresser Basalt
- Benz and Asphaug, 1994

Low strain rate
High strain rate

(From Asphaug)
Damage and degradation leading to ultimate failure occur at some limiting strain.
A Grady Kipp Implementation in 3D

• Damage is isotropic, so that when a crack is formed in one direction, all directions lose stiffness.

• As damage accumulates, the stiffness in both tension and in shear decrease, eventually to zero.

• Therefore, material failed by the outgoing shock behaves as water.

• Calibrated to disruption test, by adjusting the strength (Weibull) parameters.
Fragmentation phase: principles

Equation of state
\[ P = f(E, \rho) \]

Model of brittle Failure

Stress tensor
\[ \sigma_{\alpha\beta} = -P \delta_{\alpha\beta} + S_{\alpha\beta} \]

Conservation equations

Yielding criterion:
\[ S_{\alpha\beta} \rightarrow f S_{\alpha\beta} \]

SPH techniques
Validation with impact experiments on basalt

→ SPH simulations using $3.5 \times 10^6$ particles

Nakamura & Fujiwara 93

Benz & Asphaug 1994
High-res. Runs by M. Jutzi

largest fragment as a function of impact angle
Fragmentation Phase

Shock wave Propagation

Impact velocity: 5 km/s

Impact angle: 45°

P. Michel & W. Benz
Why porosity is important

Many (most?) asteroids and comets are porous.

Ref: Consolmagno, Britt
Internal structure

- size of computational element

- the internal structure will determine the ability to survive an impact
- the structure within some depth will determine
  - size and geometry of crater
  - amount of ejected matter
  - velocity of ejected matter
- momentum transfer
Two types of porosity:

- **Macroscopic scale:**
  - Void sizes can be modeled explicitly
  - Rock components are not porous and their fragmentation is driven by classic model of brittle failure of non-porous material

- **Microscopic scale:**
  - Void/pore sizes are smaller than the thickness of the shock front
  - Void/pore sizes are smaller than the numerical resolution
  - Fragmentation modeled using the so-called P-\(\alpha\) (Herrmann 1968) or \(\varepsilon-\alpha\) or \(\rho-\alpha\) model

\[\Rightarrow\text{assumes uniform and homogeneous porosity...}\]
Volume of voids: $V_v$
Volume of matter: $V_s$
Total: $V = V_s + V_v$
Void ratio (Porosity): $\phi = \frac{V_v}{V}$
Solid ratio: $\beta = \frac{V_s}{V} = 1 - \phi$
Distension: $\alpha = \frac{V}{V_s} = \frac{1}{1-\phi}$

Mass of solids: $m_s$
Density of mixture: $\rho = \frac{m_s}{V}$
Density of solid: $\rho_s = \frac{m_s}{V_s}$
Distension: $\alpha = \frac{\rho_s}{\rho}$
Porosity: $\phi = 1 - \frac{1}{\alpha}$
Type of porosity:

• macroscopic scale: modeled explicitly using the classical model of brittle fail.
• microscopic scale: modeled using the so-called P-α model (Herrmann 1968)
  → assumes uniform and homogeneous porosity...

Definition:

→ porosity: \[ \phi = \frac{V - V_S}{V} \rightarrow \frac{V_V}{V} \]
  With \( V_V \): Volume of voids
  \( V_S \): Volume of matrix
  \( V \): total volume

→ distension: \[ \alpha = \frac{\rho_s}{\rho}, \quad 1 \leq \alpha \leq \alpha_0 \]
  \( \rho_s \): density of matrix
  \( \rho \): bulk density

\[ \phi = 1 - \frac{1}{\alpha} \]
Distention is defined as a function of pressure:
\( \alpha = \alpha(P) \); but it can also be defined as a function of density or strain
How a porous material responds to loading... Distension as a function of pressure: A $p$-$\alpha$ description
Distention is used to modify the following equations:

→ equation of state:  \[ P \rightarrow \frac{1}{\alpha} P(\alpha \rho, u, \ldots) = \frac{1}{\alpha} P(\rho_s, u, \ldots) \]

→ deviatoric stresses:  \[ S_{ij} \rightarrow S_{ij}(\ldots, \alpha) \]

→ fracture model:  \[ D \rightarrow D(\ldots, \alpha) \]

Time evolution of distention:

\[ \dot{\alpha}(t) = \frac{\dot{u} \left( \frac{\partial P_s}{\partial u} \right) + \alpha \dot{\rho} \left( \frac{\partial P_s}{\partial \rho_s} \right)}{\alpha + \frac{d\alpha}{dP} \left[ P - \rho \left( \frac{\partial P_s}{\partial \rho_s} \right) \right]} \cdot \frac{d\alpha}{dP} \]
As the pores are crushed, the material is slowly turned into sand (at the scale of the numerical resolution element).

Since both damage $D$ and distension $\alpha$ are volume ratios, we can relate the two by (linear relation)

$$D = 1 - \frac{(\alpha - 1)}{\left(\alpha_0 - 1\right)}$$

Time evolution:

$$\frac{dD^{1/3}}{dt} = -\frac{1}{3} \left[1 - \frac{\alpha - 1}{\alpha_0 - 1}\right]^{-\frac{2}{3}} \frac{1}{\alpha_0 - 1} \frac{d\alpha}{dt}$$

*total damage = tension damage (Weibull flaws) + compression damage (breaking pores)*
First simulations of an impact experiment on a porous target (pumice)

Jutzi, Michel, Hiraoka, Nakamura, Benz, 2009, Icarus 201

Different kinds of porosity

Damage propagation (red) from the numerical simulation with porosity model

Initial material properties are those measured for the real target

Impact speed: 3 km/s
Confrontation simulations/experiments

Jutzi, Michel, Hiraoka, Nakamura, Benz, 2009, Icarus 201

Experiment $T = 1.5$ ms Simulation

First validations of a model of fragmentation of porous body
Confrontation simulation/experiment

First application at large scale: formation of the crater on the asteroid Stein (Rosetta image)

Simulating an asteroid disruption

Requires:

1. To compute the fragmentation phase (hydrocode):

Hydrodynamical equations + model of brittle failure
⇒ Propagation of the shock wave and of cracks into the target

2. To compute the gravitational phase between the generated fragments (parallel N-body Code)

First results: Michel et al. (2001), Science Vol. 294, pp 1696-1700.
Our simulations of asteroid disruptions reproduced for the first time asteroid families and suggest that objects > km are gravitational aggregates (rubble piles)

Michel et al., *Science* 294 (2001)

Disruption outcomes and impact energies greatly depend on the initial internal structure of the impacted body

Surface and internal properties: crucial information for hazard mitigation

• Example: Mission Don Quijote: phase A studies at ESA
  (final presentation: 17-18 Avril 2007)

The momentum transfer efficiency highly depends on the (sub)surface properties (e.g., porosity, regolith properties)
Current difficulties in modeling

Mass ratio:

\[ \frac{M_p}{M_t} \approx 4.4 \times 10^{-10} \]

Max. number of SPH particles:

\[ N \approx 10^7 \]

One SPH particle

\[ \sim 225 \times M_p \]

→ We cannot simulate the whole asteroid
Current difficulties in modeling

The size of the simulated domain (half-sphere) should be larger than the size of the damaged region.

Global effects cannot be studied easily.
Initial conditions (target structures)

Target:
- half-sphere of 34 m diameter
- \(4.4 \times 10^6\) SPH particles
- spatial resolution \(\sim 15\) cm
Results: damage

Simulations after 20 ms

Red: fully damaged material

1) pre-shattered

2) micro-porous

3) macro-porous

4) macro- and micro-porous

Simulations and plots made by M. Jutzi
Results: velocity

Simulations after 20 ms
Colors: vertical velocity 0.1 to 10^3 m/s (log-scale)

Simulations and plots made by M. Jutzi
Momentum transfer

\[ \vec{P}_{\text{target}} = \vec{P}_{\text{projectile}} + \vec{P}_{\text{ejecta}} > \vec{P}_{\text{projectile}} \]
Momentum transfer

• Normalized with the momentum of the projectile:

\[ P_{target} = 1 + P_{ejecta} \equiv \beta \geq 1 \]

• Change of the target velocity

\[ \Delta V = \frac{P_{target}}{M_{target}} = \beta \times \frac{P_{projectile}}{M_{target}} \]
Momentum transfer

- Momentum multiplication factor

\[ \beta \sim \left( \frac{\rho U^2}{Y} \right)^{(3\mu - 1)/2} \]

- Target structure
- Material characteristics
- Impact velocity
- Target size etc.

*from scaling laws:*
Cumulative momentum distribution

Escape velocity

\[ \Rightarrow \text{Momentum multiplication factor } \beta \]
Velocity change (of a 1 km asteroid)

ΔV is given by $\beta \times \frac{mU}{M}$

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$\beta$</th>
<th>$\Delta V$ (µm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.13</td>
<td>2.8</td>
</tr>
<tr>
<td>2</td>
<td>1.74</td>
<td>2.3</td>
</tr>
<tr>
<td>3</td>
<td>1.48</td>
<td>2.0</td>
</tr>
<tr>
<td>4</td>
<td>1.27</td>
<td>1.7</td>
</tr>
</tbody>
</table>

1: pre-shattered  
2: micro-porous  
3: macro-porous  
4: macro- and micro-porous
Laboratory Impact Disruption
Granular material with cohesion

Target

- Glass beads arranged in three layers to form a disk.
- “Sintered” in oven.
- Bond strength controlled by cooking duration.
- 90 beads total (each are $\frac{3}{16}$ inches across, 2.5 g/cc).

Initial Impact Trials

- Projectile is single glass bead $\frac{1}{8}$ inches in diameter.
- Shot from gas gun at 277 m/sec.
- Impacts near center of target at a 45° angle.
Building a Computational Model

\( N \)-body code (pkdgrav) is used to simulate forces between particles:

- Gravity
- Collisions
- Strength
  - Elastic Deformation equivalent to Hooke’s Law (springs)
Elastic (Springs) Model

- Neighboring particles “connected” by springs.
- Each spring is defined by:
  - An equilibrium separation (length at zero strain)
  - A Young’s modulus
  - A maximum stress/strain beyond which spring breaks
  - A damping term
Building the Target in Stages

• STEP ONE: Placement of bottom layer and outside middle layer atop a wall.

• STEP TWO: Adjust to avoid overlaps and attach springs, drop in remaining beads that will comprise the rest of middle layer by introducing uniform gravity (self-gravity is off).

• STEP THREE: Introduce (translucent) wall that pushes middle layer into configuration.

• STEP FOUR: Drop top layer on top.
Dependence on Young’s Modulus

Number of Particles

Young’s Modulus (GPa)

Particles in Groups of 3 or More
Particles in Groups of Two
Free Particles
Thank you!

Porous versus non-porous!