High Frequency Wave Propagation and Discrete Geodesics

Vladimir Oliker
Department of Mathematics and Computer Science
Emory University, Atlanta, Ga
oliker@mathcs.emory.edu

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• Statement of the Problem

• UTD, PTD and computational models

• DOVA and C-DOVA

• From DIFFRACTION to GEOMETRY

• Computing Geodesics

• Validations

• Summary
A principal problem in design of antenna systems mounted on platforms such as aircraft, satellites, and ships is to determine the optimal location of antennas on the platform so that the desired radiation coverage is achieved. Similarly, the problem of optimal location of antennas arises when multiple antennas are mounted on the platform and mutual interference must be minimized.

The documentation supplied by antenna manufacturers contains a description of antenna pattern when the antenna is operating in space or on a ground plane.
In fact, when antenna is mounted on a platform, such as a modern aircraft, the net antenna pattern is impacted dramatically by the complex geometry of the platform.

Consequently, to ensure proper functioning of an antenna system mounted on a platform it is critically important to determine the effects due to the platform.
To determine, at least approximately, the actual antenna performance the following approaches have been used:
1. **Build a scale model and perform measurements**

   (i) This approach is widely used and, generally, considered reliable but

   (ii) It is expensive

   (iii) It is time consuming

   (iv) Faithfulness of scaled models is not always adequate

   (v) An experimental study of antenna performance by physically changing the antenna location is difficult
2. Perform Computer Simulations

   (i) This approach is considered to be inexpensive and flexible

   (ii) It is important to know the reliability and limitations of codes used

   (iii) Accurate representation of the platform as an electronic model suitable for high frequency computations is a critical issue

3. A combination of the above two approaches
In this talk only the problem of calculating patterns of airborne antennas is discussed.

However, the same or similar issues have to be resolved for any platform and in the interference problem.
Two High Frequency Techniques

The Uniform Theory of Diffraction (UTD) and Physical Theory of Diffraction (PTD) have been widely used in EM community to deal with the above problems computationally.
UTD vs. PTD

- UTD is based on analysis of propagation paths from source to the observation point with subsequent application of appropriate diffraction mechanisms. It allows to combine diffraction effects such as

  ... + Creeping + Edge Diffraction + Spatial + Creeping + ...

- The major difficulty is the determination of propagation paths in the presence of a scatterer with complex geometry and singularities
Consequently, the standard UTD-based codes (such as the Numerical Electromagnetic Code - Basic Scattering Code (NEC-BSC) and the Radiation Pattern Code require that platforms be represented as a union of a small number of simple shapes such as cylinders, cones, plates, and ellipsoids;
see


and other references there.
Some extensions which allow quadric cylinders and quadric surfaces of revolution have also been considered; see


**Simplified platform modeling leads to significant loss of accuracy**
A simplified computer model
Here is a model generated with computer aided design (CAD) software
A CAD model
UTD vs. PTD

- PTD is based on shooting and bouncing rays with subsequent surface integration; it allows to deal with multiple bounces (if one can trace efficiently multiple reflections...)

- Typically, PTD codes do not deal with creeping-wave mechanisms which play a dominant role when the field point is in the shadow region relative to the position of antenna.
The fundamental principle of UTD and its extensions is **LOCALIZATION**

- The GTD, UTD and their extensions provide a highly developed and very sophisticated set of prescriptions for computing local diffraction coefficients (=effects).

- Therefore, if the propagation paths from the source to the field point are known then by “gluing” together along the paths the local diffraction data it is possible to compute the resulting field.
COMMENTS:

- Determination of propagation paths on a general surface is a problem of global geometry

- The GTD/UTD,... are local theories and do not provide means for finding propagation paths

- Determination of propagation paths on faceted surfaces (or NURBS) is also a problem of computational geometry
THE MAIN ISSUES

• Determine propagation paths
• Obtain geometrical data
• Identify UTD mechanisms and apply UTD coefficients
• Implement and integrate into an industrial-strength user-friendly code
The DOVA and C-DOVA CODES

During 1994 - 2002, with the support from AFOSR, SBIR, and other sources there have been developed and implemented in software codes new geometric methods for calculating propagation paths over fully realistic platform models, represented as a mesh of triangular facets generated with widely used commercial CAD packages.

These geometric methods were integrated with UTD techniques into DOVA (= Diffraction Over Virtual Airframe) and C-DOVA codes. DOVA calculates antenna radiation patterns and C-DOVA calculates EM coupling. Both are industrial-strength and user-friendly codes.

Earlier and current versions of these codes have been in operational use by US defense contractors since late ’90th.
Some References


Propagation Path
Finding Propagation Paths; Variational Formulation

Denote by $S$ the surface of a scatterer. It is assumed to be a closed, embedded, and oriented polyhedral surface in $\mathbb{R}^3$, triangulated so that any edge belongs to exactly two triangles and the intersection of any two triangles is either an edge, a vertex, or empty. The orientation is always assumed to be outward. Note that $S$ is not assumed to be convex.

Denote by $\Omega$ the open subset of $\mathbb{R}^3$ bounded by $S$. Let $\gamma$ be a curve given by a piece-wise linear vector function

$$
\gamma(\sigma) = (x(\sigma), y(\sigma), z(\sigma)), \quad \sigma \in [0, \infty),
$$

(1)

where $\sigma$ is the arc length.
Finding Propagation Paths; Variational Formulation

The Fermat functional is given by

\[ F(\gamma_0, \gamma_1) = \int n d\sigma, \]

where \( n = \text{const} \) is the refractive index, \( d\sigma \) is the arc length, and the integral is taken over the piece of \( \gamma \) between points \( \gamma_0 \) and \( \gamma_1 \).

\( T \) is the radiation source located on or off \( S \).

The minimizers of \( F \) (in appropriate classes) are also pieces of geodesics on \( S \) and in space.
A curve (1) is “admissible” for $\mathcal{F}$ if:

\begin{align*}
\gamma & \text{ is simple,} \\
\gamma(0) & = T, \\
\frac{d\gamma}{d\sigma} & \to \hat{f} \quad \text{as} \quad \sigma \to \infty,
\end{align*}

where $\hat{f}$ is a far-field direction, and

\begin{equation}
\gamma(\sigma) \cap \Omega = \emptyset \quad \text{for any} \quad \sigma \in [0, \infty).
\end{equation}

Let $\mathcal{R}(S, T, \hat{f})$ be the collection of admissible curves for the scatterer $S$, source $T$, and far-field direction $\hat{f}$. A path $\gamma \in \mathcal{R}$ is stationary for the Fermat functional $\mathcal{F}$ if

\begin{equation}
\delta \mathcal{F} = 0,
\end{equation}

where $\delta$ is the variation of $\mathcal{F}$ in $\mathcal{R}(S, T, \hat{f})$.

——PROBLEM STATEMENT——

For a given scatterer $S$, a point source $T$ on or off the scatterer $S$, and a given far-field direction $\hat{f}$ find the stationary paths of the Fermat functional in the class $\mathcal{R}(S, T, \hat{f})$.

——NOTE——

To capture paths which diffract at corners, edges (in certain cases), or reflect it is necessary to consider CONSTRAINED (C-) variations.
MAIN LOOP

1. Initialize loop over far-field directions
2. Determine the direct paths
3. Create initial paths for a given far-field direction
4. Optimize each of the initial paths
5. Compute UTD fields
6. If the loop over far-field directions is not complete increment the index of the far-field direction and go to 2; otherwise, exit
PATH/SURFACE GEOMETRY DATA - 1

• Special algorithms are developed for “extracting” from faceted files $C^4$ and $C^2$ data such as
• surface normals and tangents
• principal curvatures, principal directions, geodesic curvature, torsion, Gaussian curvature, etc.
FOCK PARAMETER

\[ \xi = \left( \frac{k}{2} \right)^{1/3} \int_{\gamma} \frac{dt'}{\rho^{2/3}} \]

WHERE

\[ \rho_g(t) = |d\tau(t)/dt|^{-1} \]
### Examples of Fock Parameter Computations by DOVA-P

<table>
<thead>
<tr>
<th>Test #</th>
<th>Surface</th>
<th>Grid</th>
<th>Path Length</th>
<th>Fock Parameter Value</th>
<th>Error</th>
</tr>
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<tbody>
<tr>
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<td></td>
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<td>Calculated by DOVA</td>
<td>Analytically Predicted</td>
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<tr>
<td>1</td>
<td>Sphere of Radius 1</td>
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<td>Divergence Factor Value</td>
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<td>2.048558</td>
<td>2.043845</td>
</tr>
</tbody>
</table>
UTD FIELD COMPUTATION

- ANALYZE PATH TO IDENTIFY UTD MECHANISMS
- APPLY APPROPRIATE WAVE-SPREADING, REFLECTION/DIFFRACTION COEFFS., FOCK FUNCTIONS, ETC.
- ACCEPT/REJECT
- ADD CONTRIBUTION FOR ACCEPTED PATH TO TOTAL FOR GIVEN FAR-FIELD DIRECTION
Current Capabilities of DOVA

Based on user’s choice of the

- aircraft model,
- antenna type,
- antenna position,
- antenna physical characteristics, and
- pattern cut,
the DOVA system

- builds propagation paths from radiating source to given observation points in the far-field,

- classifies diffraction mechanisms and computes all geometric characteristics required for electromagnetic analysis,

- computes and integrates field contributions by different diffraction mechanisms,

- displays computed results graphically.
Currently, the following diffraction mechanisms are identified and analyzed:

(i) direct paths propagating in space,

(ii) surface diffraction ("creeping" waves),

(iii) wedge diffraction,

(iv) double-edge diffractions,

(v) reflection,

(vi) propagation mechanisms which are combinations of (i) - (v).
Diffraction and Geometry

Finding Propagation Paths
Validation

Accuracy of computations has been tested against almost any published result that could be found as well as against available data from measurements, including

- A comparison with results for canonical surfaces

- A comparison with results obtained by measurements on scaled models or calculated by other methods.
Fig. 48

Fig. 48. The $|E|$ radiation pattern in the $xz$ plane of a circumferential $x$-directed slot in a sphere. (After Pathak, Wang, Burnside, and Kouyoumjian [31], © 1991 IEEE)

DOVA results:

Figure 12: The $|E|$ radiation patterns in the $xz$ plane of the circumferential $X$-directed slot in a sphere.
Fig. 52c

New York: Chapman & Hall, 1993

DOVA RESULTS:

$\theta = 100$

1. Magnetic rectangular slot with uniform current
2. Electric current
3. Antenna location
4. Antenna length
5. Antenna width
6. Antenna polarization: $x$-directed
7. Antenna wavelength ($\lambda = 7.5$)
8. Plane cut: horizontal
9. Elevation: $-10$ to $90$ steps
10. Min max steps: $-10$ to $40$
Fig. 50

P.H Pathak,
"Techniques in
High-Frequency
Problems" in
The Antenna Handbook,
Ed. by Y.T. Lo & S.W. Lee
New York:
Chapman & Hall, 1993

DOVA RESULTS:

\[ \theta = 80 \]

<table>
<thead>
<tr>
<th>Magnetic rectangular slot with uniform current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension:</td>
</tr>
<tr>
<td>Length:</td>
</tr>
<tr>
<td>Width:</td>
</tr>
<tr>
<td>Radial polarization:</td>
</tr>
<tr>
<td>Wave number:</td>
</tr>
<tr>
<td>Plane cut:</td>
</tr>
<tr>
<td>Elevation:</td>
</tr>
</tbody>
</table>

E_\theta

\[
\begin{align*}
E_\theta & = Q = 80 \\
0 & 180 80 \# \text{min max steps}
\end{align*}
\]
SUMMARY

- With new ray tracing techniques it has become possible to analyze antenna patterns and antenna interferences on CAD-based models which provide a faithful representation of actual platforms.

- The developed algorithms and codes have been successfully tested on numerous examples and have been used in production since 1999.

- The developed algorithms and codes perform the required ray tracing and calculations in nearly real time on desktop computers.