Introduction to Laser Doppler Velocimetry

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Laser Doppler Anemometry (LDA)

- Single-point optical velocimetry method

Study of the flow between rotating impeller blades of a pump

3-D LDA Measurements on a 1:5 Mercedes-Benz E-class model car in wind tunnel
Phase Doppler Anemometry (PDA)

- Single point particle sizing/velocimetry method

Drop Size and Velocity measurements in an atomized Stream of Moleten Metal

Droplet Size Distributions Measured in a Kerosene Spray Produced by a Fuel Injector
Laser Doppler Anemometry

- **LDA**
  - A high resolution - single point technique for velocity measurements in turbulent flows

A Back Scatter LDA System for One Velocity Component Measurement (Dantec Dynamics)

- **Basics**
  - Seed flow with small tracer particles
  - Illuminate flow with one or more coherent, polarized laser beams to form a MV
  - Receive scattered light from particles passing through MV and interfere with additional light sources
  - Measurement of the resultant light intensity frequency is related to particle velocity
LDA in a nutshell

• **Benefits**
  – Essentially non-intrusive
  – Hostile environments
  – Very accurate
  – No calibration
  – High data rates
  – Good spatial & temporal resolution

• **Limitations**
  – Expensive equipment
  – Flow must be seeded with particles if none naturally exist
  – Single point measurement technique
  – Can be difficult to collect data very near walls
Review of Wave Characteristics

- General wave propagation

\[ \psi(x, t) = A \cos \left( 2\pi \left( \frac{x}{\lambda} - \frac{t}{\tau} \right) \right) \]

\[ \psi(x, t) = \text{Re} \left\{ A e^{i[kx - \omega t + \phi]} \right\} \]

- \( A \) = Amplitude
- \( k \) = wavenumber
- \( x \) = spatial coordinate
- \( t \) = time
- \( \omega \) = angular frequency
- \( \varepsilon \) = phase

\[ k = \frac{2\pi}{\lambda} \quad \tau = \frac{\lambda}{c} \quad \omega = \frac{2\pi}{\tau} = \frac{2\pi c}{\lambda} \]
Electromagnetic waves: coherence

• Light is emitted in “wavetrains”
  – Short duration, $\Delta t$
  – Corresponding phase shift, $\varepsilon(t)$; where $\varepsilon$ may vary on scale $t > \Delta t$
    \[
    E = E_o \exp \left[ i(kx - \omega t + \varepsilon(t)) \right]
    \]

• Light is *coherent* when the phase remains constant for a sufficiently long time
  – Typical duration ($\Delta t_c$) and equivalent propagation length ($\Delta l_c$) over which some sources remain coherent are:

<table>
<thead>
<tr>
<th>Source</th>
<th>$\lambda_{\text{nom}}$ (nm)</th>
<th>$\Delta l_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>White light</td>
<td>550</td>
<td>8 $\mu$m</td>
</tr>
<tr>
<td>Mercury Arc</td>
<td>546</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>Kr$^{86}$ discharge lamp</td>
<td>606</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Stabilized He-Ne laser</td>
<td>633</td>
<td>$\leq 400$ m</td>
</tr>
</tbody>
</table>

– Interferometry is only practical with coherent light sources
Electromagnetic waves: irradiance

• Instantaneous power density given by Poynting vector
  – Units of Energy/(Area-Time)

\[ \mathbf{S} = c^2 \varepsilon_0 \mathbf{E} \times \mathbf{B} \]

\[ S = c \varepsilon_0 E^2 \]

• More useful: average over times longer than light freq.

\[ \langle f \rangle_T = \frac{1}{T} \int_{t-T/2}^{t+T/2} f(t') dt' \]

\[ I = \langle S \rangle_T = c \varepsilon_0 \langle E^2 \rangle_T = \frac{c \varepsilon_0}{2} \mathbf{E} \cdot \mathbf{E}^* = \frac{c \varepsilon_0}{2} E_0^2 \]
LDA: Doppler effect frequency shift

- Overall Doppler shift due two separate changes
  - The particle ‘sees’ a shift in incident light frequency due to particle motion
  - Scattered light from particle to stationary detector is shifted due to particle motion
LDA: Doppler shift, effect I

• **Frequency Observed by Particle**
  – The first shift can itself be split into two effects
    • (a) the number of wavefronts the particle passes in a time $\Delta t$, as though the waves were stationary…

$$ \hat{e}_b = \frac{k}{\|k\|} $$

Number of wavefronts particle passes during $\Delta t$ due to particle velocity:

$$ \frac{u \cdot \hat{e}_b \Delta t}{\lambda} $$
LDA: Doppler shift, effect I

- Frequency Observed by Particle
  - The first shift can itself be split into two effects
    - (b) the number of wavefronts passing a stationary particle position over the same duration, \( \Delta t \). . .

\[ \hat{e}_b = \frac{k}{|k|} \]

Number of wavefronts that pass a stationary particle during \( \Delta t \) due to the wavefront velocity:

\[ \frac{c \Delta t}{\lambda} \]
LDA: Doppler shift, effect I

- The net effect due to a moving observer w/ a stationary source is then the difference:

\[
\Delta t = \frac{c\Delta t}{\lambda} - \frac{\mathbf{u} \cdot \hat{e}_b \Delta t}{\lambda}
\]

Number of wavefronts that pass a moving particle during \(\Delta t\) due to combined velocity (same as using relative velocity in particle frame):

\[
f_p = \frac{\text{# of wavefront s}}{\Delta t}
= \frac{c}{\lambda} \left(1 - \frac{\mathbf{u} \cdot \hat{e}_b}{c}\right)
= f_0 \left(1 - \frac{\mathbf{u} \cdot \hat{e}_b}{c}\right)
\]
LDA: Doppler shift, effect II

- An additional shift happens when the light gets scattered by the particle and is observed by the detector
  - This is the case of a moving source and stationary detector (classic train whistle problem)

Distance a scattered wave front would travel during $\Delta t$ in the direction of detector, if $\mathbf{u}$ were 0:

Due to source motion, the distance is changed by an amount:

Therefore, the effective scattered wavelength is:

$$
\lambda_s = \frac{\text{net distance traveled by wave}}{\text{number of waves emitted}} = \frac{c\Delta t - \mathbf{u} \cdot \hat{\mathbf{e}}_s \Delta t}{f_p \Delta t} = \frac{c - \mathbf{u} \cdot \hat{\mathbf{e}}_s}{f_p}
$$
LDA: Doppler shift, I & II combined

- Combining the two effects gives:

\[
f_{\text{obs}} = \frac{c}{\lambda_s} = \frac{cf_p}{c - u \cdot \hat{e}_s} = \frac{f_p}{1 - \frac{u \cdot \hat{e}_s}{c}} = f_0 \left( \frac{1 - \frac{u \cdot \hat{e}_b}{c}}{1 - \frac{u \cdot \hat{e}_s}{c}} \right)
\]

- For \( u \ll c \), we can approximate

\[
f_{\text{obs}} = f_0 \left(1 - \frac{u \cdot \hat{e}_b}{c}\right) \left(1 - \frac{u \cdot \hat{e}_s}{c}\right)^{-1}
\]

\[
= f_0 \left(1 - \frac{u \cdot \hat{e}_b}{c}\right) \left[1 + \frac{u \cdot \hat{e}_s}{c} - \left(\frac{u \cdot \hat{e}_s}{c}\right)^2 + \cdots\right]
\]

\[
= f_0 \left(1 + \frac{1}{c} u \cdot \hat{e}_s - \hat{e}_b \right) \cdots
\]

\[
\cong f_0 + \frac{f_0}{c} u \cdot \hat{e}_s - \hat{e}_b
\]
LDA: problem with single source/detector

- **Single beam frequency shift depends on:**
  - velocity magnitude
  - Velocity direction
  - observation angle

  \[ f_{obs} \approx f_0 + \frac{f_0}{c} \mathbf{u} \cdot \mathbf{e}_s - \mathbf{e}_b \]

- **Additionally, base frequency is quite high…**
  - \( O[10^{14}] \text{ Hz} \), making direct detection quite difficult

- **Solution?**
  - Optical heterodyne
    - Use interference of two beams or two detectors to create a “beating” effect, like two slightly out of tune guitar strings, e.g.

    \[
    \cos[\omega_1 t] \cos[\omega_2 t] = \frac{1}{2} \left( \cos[(\omega_1 + \omega_2) t] + \cos[(\omega_1 - \omega_2) t] \right)
    \]

    - Need to repeat for optical waves

      \[
      \mathbf{E}_1 = E_{o1} \cos \mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t
      \]

      \[
      \mathbf{E}_2 = E_{o2} \cos \mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t
      \]
Optical Heterodyne

• Repeat, but allow for different frequencies...

\[ I = \frac{c\varepsilon_0}{2} (E_1 + E_2) \cdot (E_1^* + E_2^*) \]

\[ E_1 = E_{01} \exp[i(k_1x - \omega_1t + \varepsilon_1)] = E_{01} \exp[i\phi_1] \]

\[ E_2 = E_{02} \exp[i(k_2x - \omega_2t + \varepsilon_2)] = E_{02} \exp[i\phi_2] \]

\[ I = \frac{c\varepsilon_0}{2} \left[ E_{o1}^2 + E_{o2}^2 + E_{o1} \exp(i\phi_1)E_{o2} \exp(-i\phi_2) + E_{o1} \exp(-i\phi_1)E_{o2} \exp(i\phi_2) \right] \]

\[ I = \frac{c\varepsilon_0}{2} \left[ E_{o1}^2 + E_{o2}^2 + 2E_{o1}E_{o2} \left\{ \frac{\exp(i(\phi_1 - \phi_2)) + \exp(-i(\phi_1 - \phi_2))}{2} \right\} \right] \]

\[ I = \frac{c\varepsilon_0}{2} \left[ E_{o1}^2 + E_{o2}^2 + 2E_{o1}E_{o2} \cos(\phi_1 - \phi_2) \right] \]

\[ I = \frac{c\varepsilon_0}{2} \left[ E_{o1}^2 + E_{o2}^2 + 2E_{o1}E_{o2} \cos[(k_1 - k_2) \cdot r - (\omega_1 - \omega_2)t + (\varepsilon_1 - \varepsilon_2)] \right] \]

\[ = \frac{1}{2} \left[ I_{o1} + I_{o2} + 2\sqrt{I_{o1}I_{o2}} \cos[(k_1 - k_2) \cdot r - (\omega_1 - \omega_2)t + (\varepsilon_1 - \varepsilon_2)] \right] \]

\[ I_{PED} + I_{AC} \]
How do you get different scatter frequencies?

• For a single beam

\[ f_s \approx f_0 + \frac{f_0}{C} \mathbf{u} \cdot \mathbf{e}_s - \hat{\mathbf{e}}_b \]

  – Frequency depends on directions of \( \mathbf{e}_s \) and \( \mathbf{e}_b \)

• Three common methods have been used
  – Reference beam mode (single scatter and single beam)
  – Single-beam, dual scatter (two observation angles)
  – Dual beam (two incident beams, single observation location)
Dual beam method

Real MV formed by two beams
Beam crossing angle $\gamma$
Scattering angle $\theta$

‘Forward’ Scatter Configuration
Dual beam method (cont)

\[ f_{s,1} = f_0 + \frac{f_0}{c} \mathbf{u} \cdot \mathbf{e}_{s,1} - \mathbf{e}_{b,1} \]
\[ f_{s,2} = f_0 + \frac{f_0}{c} \mathbf{u} \cdot \mathbf{e}_{s,2} - \mathbf{e}_{b,2} \]
\[ \therefore \Delta f = \frac{f_0}{c} \mathbf{u} \cdot \mathbf{e}_{b,2} - \mathbf{e}_{b,1} \]

Note that \((\mathbf{e}_{b,1} - \mathbf{e}_{b,2}) = 2\sin(\gamma / 2) \mathbf{\hat{x}}_g\)

So:
\[ \mathbf{u} \cdot \mathbf{x}_g = \frac{\lambda}{2\sin(\zeta / 2)} \Delta f_D \]

Measure the component of \(\mathbf{u}\) in the \(\mathbf{\hat{x}}_g\) direction

\[ I = \frac{1}{2} \left[ I_{o1} + I_{o2} + 2\sqrt{I_{o1}I_{o2}} \cos \left( \mathbf{e}_1 - \mathbf{k}_2 \right) \mathbf{r} - \left( \frac{4\pi \sin(\zeta / 2)}{\lambda} \mathbf{u} \cdot \mathbf{x}_g \right) t + \mathbf{e}_1 - \mathbf{e}_2 \right] \]
**Fringe Interference description**

- **Interference “fringes” seen as standing waves**
  - Particles passing through fringes scatter light in regions of constructive interference

\[ \Lambda = \frac{\lambda}{2 \sin \left( \frac{\nu}{2} \right)} = \frac{\mathbf{u} \cdot \mathbf{x}_s}{\Delta f} \]

- Adequate explanation for particles smaller than individual fringes
Gaussian beam effects

- Power distribution in MV will be Gaussian shaped
- In the MV, true plane waves occur only at the focal point
- Even for a perfect particle trajectory the strength of the Doppler ‘burst’ will vary with position

Figures from Albrecht et. al., 2003
Non-uniform beam effects

- Off-center trajectory results in weakened signal visibility
- Pedestal (DC part of signal) is removed by a high pass filter after photomultiplier

Figures from Albrecht et. al., 2003
Multi-component dual beam

Three independent directions

Two-component probe Looking Toward the Transmitter
Sign ambiguity...

- Change in sign of velocity has no effect on frequency

\[ I = \left| I_0 + 2I_0 \cos \left( k_1 - k_2 \cdot r - 2\pi f_D t + \varphi_1 - \varphi_2 \right) \right| \]

\[ \mathbf{u} \cdot \mathbf{x}_s = \frac{\lambda}{2 \sin \varphi / 2} \Delta f_D \]

Graph shows waveforms for beams 1 and 2 with different sign of \( u_{xg} \).
Velocity Ambiguity

- **Equal frequency beams**
  - No difference with velocity direction... cannot detect reversed flow
- **Solution: Introduce a frequency shift into 1 of the two beams**

\[
\begin{align*}
  f_{b} &= 5.8 \times 10^{14} \\
  f_{b1} &= f_{b} \\
  f_{b2} &= f_{\text{bragg}} + f_{b}
\end{align*}
\]

\[
\begin{align*}
  f_{s,1} &= f_{b} + \frac{f_{b}}{c} \mathbf{u} \cdot (\hat{e}_{s,1} - \hat{e}_{b,1}) \\
  f_{s,2} &= (f_{b} + f_{\text{bragg}}) + \frac{f_{b}}{c} \mathbf{u} \cdot (\hat{e}_{s,2} - \hat{e}_{b,2}) \\
  \Delta f_{D} &= f_{\text{bragg}} + \frac{f_{b}}{c} \mathbf{u} \cdot (\hat{e}_{b,1} - \hat{e}_{b,2}) = f_{\text{bragg}} + \Delta f_{D0}
\end{align*}
\]

**New Signal**

\[
I \propto 2E_{01}^{2} \cos(-2\pi \{\Delta f_{D0} + f_{\text{bragg}}\}t)
\]

**Hypothetical shift Without Bragg Cell**

- If \(\Delta f_{D} < f_{\text{bragg}}\) then \(u < 0\)
Frequency shift: Fringe description

- Different frequency causes an apparent velocity in fringes
  - Effect result of interference of two traveling waves as slightly different frequency
Directional ambiguity (cont)

\[ u_{xg} = \frac{\lambda (\Delta f_{D0} - f_{\text{bragg}})}{2 \sin(\gamma / 2)} \]

\[ \lambda = 514 \text{ nm}, f_{\text{bragg}} = 40 \text{ MHz} \text{ and } \gamma = 20^\circ \]

Upper limit on positive velocity limited only by time response of detector
Velocity bias sampling effects

- **LDA samples the flow based on**
  - Rate at which particles pass through the detection volume
  - Inherently a flux-weighted measurement
  - Simple number weighted means are biased for unsteady flows and need to be corrected

- **Consider:**
  - Uniform seeding density (# particles/volume)
  - Flow moves at steady speed of 5 units/sec for 4 seconds (giving 20 samples) would measure:
    \[
    \frac{5 \times 20}{20} = 5
    \]
  - Flow that moves at 8 units/sec for 2 sec (giving 16 samples), then 2 units/sec for 2 second (giving 4 samples) would give
    \[
    \frac{16 \times 8 + 4 \times 2}{20} = 6.8
    \]
Laser Doppler Anemometry
Velocity Measurement Bias

Mean Velocity

\[
\bar{U}_x = \frac{\sum_{i=1}^{N} U_{x,i} \tau_i}{\sum_{i=1}^{N} \tau_i}
\]

Bias Compensation Formulas

\[
\bar{U}_x^n = \frac{\sum_{i=1}^{N} U_{x,i} - \bar{U}_x^n \tau_i}{\sum_{i=1}^{N} \tau_i}
\]

n\text{th moment}

- The sampling rate of a volume of fluid containing particles increases with the velocity of that volume
- Introduces a bias towards sampling higher velocity particles
Phase Doppler Anemometry

The overall phase difference is proportional to particle diameter

\[ \Delta \varepsilon = \frac{2\pi n_i D}{\lambda} \beta(\theta, \psi, \gamma, n_p, n_i) \]

The geometric factor, \( \beta \)
- Has closed form solution for \( p = 0 \) and \( 1 \) only
- Absolute value increases with \( \psi \) (elevation angle relative to 0°)
- Is independent of \( n_p \) for reflection