MATH 141H Exam 3 Preparation

Office Hours
(1) Tuesday & Thursday : 9am – 4pm
(2) Wednesday: 2 – 4pm
(3) Friday: 9 – 10:30 am
(4) Other times are available by appointment.

Office: 4123 CSIC, Email: hbae@cscamm.umd.edu

1. Integration by Parts
(1) Integration by Parts.  (2) Substitution + Integration by Parts.

2. Trigonometric Integrals
(1) Integrals of the form \( \int [\sin^m x \cos^n x] \, dx \), \( \int [\tan^m x \sec^n x] \, dx \)
(2) NO reduction formulas

3. Trigonometric Substitutions
(1) Integrals containing \( \sqrt{a^2 - x^2} \), \( \sqrt{a^2 + x^2} \), \( \sqrt{x^2 - a^2} \).
(2) Using triangles, convert functions of \( \theta \) to functions of \( x \) at the end.

4. Partial Fractions: Integrals of the form \( \int \frac{P(x)}{Q(x)} \, dx \).
(1) Degree of \( P \geq \) Degree of \( Q \) \( \Rightarrow \) Divide \( P \) by \( Q \) until Degree of the remainder \( R < \) Degree of \( Q \).
(2) Factorize \( R \) and \( Q \) if possible.
(3) Express \( \frac{R(x)}{Q(x)} \) as \( \frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \cdots + \frac{B_1x + C_1}{px^2 + qx + r} + \frac{B_2x + C_2}{(px^2 + qx + r)^2} + \cdots \)

5. Improper Integrals
(1) Integrals with unbounded integrands,  (2) Integrals over unbounded intervals
(3) Comparison property

6. Solve all Homework problems again.
Compute the following integrals.

1. \[ \int \frac{x^3}{\sqrt{x^2 + 9}} \, dx \]

2. \[ \int \frac{x + 4}{(x + 1)^2} \, dx \]

3. \[ \int_0^1 \frac{3x + 1}{x^2 + 3x + 2} \, dx \]

4. \[ \int \tan^3 x \sec^4 x \, dx \]

5. \[ \int \frac{1}{x^2 \sqrt{x^2 - 1}} \, dx \]

6. \[ \int \sin^3 x \cos^2 x \, dx \]

7. \[ \int \frac{1}{x^2 \sqrt{x^2 + 1}} \, dx \]

8. \[ \int \frac{x + 1}{x^2 - 3x + 2} \, dx \]
Determine whether the following integrals converge. If so, compute the integral.

1. \[\int_e^\infty \frac{1}{x \ln x} \, dx\]

2. \[\int_e^\infty \frac{1}{x (\ln x)^2} \, dx\]

3. \[\int_{-1}^\infty \frac{x}{1 + x^2} \, dx\]

4. \[\int_0^\infty \frac{1}{(x + 1) \ln x} \, dx\]

5. \[\int_1^\infty \frac{1}{x(x + 1)} \, dx\]

6. \[\int_0^\infty \frac{x}{e^{4x}} \, dx\]

Determine whether the following integrals converge using the comparison property.

1. \[\int_0^\infty \frac{x - 1}{x^3 + 3x + 2} \, dx\]

2. \[\int_0^\infty \frac{2x + \sin x}{e^{3x}} \, dx\]