MATH 141 - Final Exam - December 15, 2011

PUT EACH OF THE 9 NUMBERED PROBLEMS ON A SEPARATE ANSWER SHEET
YOUR NAME & TA’S NAME & PROBLEM NUMBER ON EACH ANSWER SHEET
SHOW YOUR WORK FOR CREDIT & NO ELECTRONIC DEVICES
COPY AND SIGN THE HONOR PLEDGE ON PAGE 1

1. (a) (15) Let \( f(x) = \sqrt{x} e^x \) and let \( R \) be the region bounded by graph of \( f \) and the \( x \)-axis on the interval \([0, 1]\). Find the volume \( V \) of the solid obtained by revolving the region \( R \) around the \( x \)-axis. You do not need to simplify your answer.

(b) (10) Find the length of the curve described parametrically by:
\( x = t \) and \( y = \ln \cos t \) for \( 0 \leq t \leq \pi/3 \). You do not need to simplify your answer.

2. (a) (10) A swimming pool has the shape of a right circular cylinder with radius 10 feet and depth 8 feet. If the pool initially contains 5 feet of water, what is the work required to pump all of the water to the top of the pool? The weight of water is 62.5 pounds per cubic foot. You do not need to simplify your answer.

(b) (10) Suppose that extending a spring 2 meters beyond its natural length requires 14 joules of work. Find the work \( W \) required to extend the spring from 2 meters beyond its natural length to 4 meters beyond its natural length.

3. (a) (10) Let
\[ f(x) = \int_0^x \cos^2(t) \, dt \]
for all \( x \). Show that \( f \) has an inverse. Let \( c = f(\pi/4) \), find \((f^{-1})'(c)\).

(b) (15) Find the particular solution of the linear differential equation that satisfies the given initial condition:
\[ \frac{dy}{dx} - \frac{2x}{x^2 + 1} y = 2x, \quad y(0) = 2. \]

4. (a) (10) Find the limit
\[ \lim_{x \to \infty} \left( 1 + \frac{1}{3x} \right)^{2x}. \]

(b) (10) Find the derivative of \((\cos x)^x\).

PLEASE TURN OVER FOR THE REST OF THE PROBLEMS

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5. (a) (15) Evaluate the indefinite integral
\[ \int \frac{1}{\sqrt{(x^2 + 1)^3}} \, dx. \]

(b) (10) Evaluate the indefinite integral
\[ \int \frac{x^2}{x^2 - 1} \, dx. \]

6. (a) (10) Determine if the following integral is convergent or divergent. If convergent, find the value.
\[ \int_0^\infty \frac{dx}{(x + 1)^{1.01}} \]

(b) (10) Does the following series converge absolutely? Give your reasoning. Name any convergence or divergence test that you are using.
\[ \sum_{n=4}^{\infty} \frac{(-1)^n}{\ln n} \]

7. (a) (15) Find the numerical value of
\[ \sum_{n=1}^{\infty} \frac{4^{n+1}}{3^{2n}}. \]

(b) (10) Does the following series converge? Give your reasoning. Name any convergence or divergence test that you are using.
\[ \sum_{n=1}^{\infty} \frac{n^2 - \sqrt{n} - 1}{n^2 - 9} \]

8. (15) Let
\[ h(t) = \frac{1}{1 + 3t^2}. \]
Using a power series that you know, give the eighth Taylor polynomial of \( h \) about 0.

9. (a) (10) Express the following complex number in the form \( a + bi \)
\[ e^{-\ln 2 + i(\pi/6)}. \]

(b) (15) Consider the circle \( r = \cos \theta \) and cardioid \( r = 1 - \cos \theta \). Sketch the graphs of both curves on the same plane. Find the area \( A \) of the region \( R \) lying outside the cardioid and inside the circle. You do not need to simplify your answer.

END OF EXAM - GOOD LUCK!