Instructions: Answer each of the 9 numbered problems on a separate answer sheet. Each answer sheet must have your name, your TA's name, and the problem number (= page number). Show all your work for each problem clearly on the answer sheet for that problem. You must show enough written work to justify your answers. Finally, please copy and sign the Honor Pledge on page 1.

NO CALCULATORS OR OTHER ELECTRONIC DEVICES

1. (a) (10) Calculate the volume $V$ of the solid generated by revolving around the $x$ axis the region $R$ bounded by the $x$-axis, the $y$-axis, the line $x = 1/2$ and the function $f$ given by $f(x) = \frac{1}{\sqrt{4x^2 + 1}}$. Sketch a graph of the region $R$.

(b) (10) Let the curve $C$ be given parametrically by $x = \sin t - t \cos t$ and $y = t \sin t + \cos t$ for $0 \leq t \leq \pi$. Find the length $L$ of $C$.

2. (a) (10) Suppose 12 joules of work are required to extend a spring from its natural length of 1 meter to 3 meters. Find the work $W$ done in extending the spring from a length of 3 meters to a length of 5 meters.

(b) (15) A tank is in the shape of the curve $y = 8x^3$ for $0 \leq x \leq 1$ revolved about the $y$ axis, with the $x$ and $y$ axes labeled in feet. Suppose the tank is filled with water weighing 62.5 lbs/ft$^3$. Draw a picture of the situation. Then write down the integral for the work $W$ required to bring the water to the top of the tank until there is a depth of 1 foot of water in the tank. Prepare the integral for integration, but DO NOT EVALUATE THE INTEGRAL.

3. (a) (10) Let $f(x) = 2e^{x^3}$. Show that $f$ has an inverse, and find $(f^{-1})'(2e)$.

(b) (10) Consider the differential equation $e^y \, dx = 2x (e^y + 1) \, dy$. Find the general solution of the differential equation and then the particular solution $y_p$ satisfying $y(2) = 0$.

4. (a) (10) Let $g(x) = \tan^{-1}(x)$. Sketch the graph of $g$, and find $g'(x)$.

(b) (10) Simplify the expression $\sin(\tan^{-1}\sqrt{w})$.

(c) (10) Let $f(t) = t^{\sin t}$. Find the domain of $f$, and then find $f'(t)$.

PLEASE TURN OVER FOR THE REST OF THE PROBLEMS
5. (a) Evaluate \( \int \frac{x^2}{(9-x^2)^{3/2}} \, dx. \)

(b) Evaluate \( \int_1^4 \sqrt{y} \ln y \, dy. \)

6. (a) Determine if \( \int_{-1}^{\infty} \frac{x}{1+x^2} \, dx \) is convergent, or divergent. If convergent, find its value.

(b) Evaluate \( \lim_{n \to \infty} (e^{2n} - 1)^{1/n} \), giving reasons.

7. (a) Find the numerical value of \( \sum_{k=2}^{\infty} \frac{2^k + 3^k}{2^k3^k} \).

(b) Determine whether the series \( \sum_{k=2}^{\infty} k^2e^{-k} \) converges or diverges. Give reasons.

8. Let \( f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2} \).

(a) Write \( f'(x) \) as a power series of the form \( \sum_{n=m}^{\infty} c_n x^n \) for appropriate values of \( c_n \) and appropriate \( m \). Also find the radius of convergence \( R \) for the power series for \( f'(x) \).

(b) Using the power series for \( f \) given in this problem, write \( \int_0^a f(t) \, dt \) as a power series.

9. (a) Find all complex solutions, in \( a + bi \) form, of the equation \( z^3 = -1 \), and sketch them on the complex plane.

(b) Consider the circle \( r = 3 \) and cardioid \( r = 2(1 + \cos \theta) \).

i. (5) Sketch the graphs of both curves on the same plane.

ii. (10) Find the area \( A \) of the region \( R \) lying inside the circle and outside the cardioid.

END OF EXAM – GOOD LUCK!