Math 141H Homework 1, Due 1/30 11am

Show all your work. Jumping to the right answer without minimum reasoning deserves no credit.

1. Find the volume of the solid whose base is the region given by the ellipse $4x^2 + y^2 = 1$ such that
   
   (1) each cross-section perpendicular to the $x$ axis is a solid square.
   
   (2) each cross-section perpendicular to the $y$ axis is a solid square.

2. Let $R$ be the region between the $x$ axis and the graph of $y = x(x^3 + 1)^{\frac{1}{4}}$ for $1 \leq x \leq 2$. Calculate the volume $V$ of the solid generated by revolving $R$ around the $x$ axis.

3. Let $R$ be the region bounded by the curve $y = x^2$ and $y = 2x$. Find the volume of the solid obtained by revolving the region $R$
   
   (1) around the $x$ axis.
   
   (2) around the $y$ axis.

4. Find the volume $V$ of the solid generated by revolving about the $x$ axis the region between the graphs of $y = \frac{1}{2}x^2 + 3$ and $y = 12 - \frac{1}{2}x^2$.

5. Let $f(x) = e^{x^2}$ and $g(x) = e^{-x^2}$. Find the volume of the solid obtained by revolving the region between the graph of $f$ and $g$ for $0 \leq x \leq 1$ around the $y$ axis.

6. Find the volume of the ellipsoid obtained by rotating the ellipse $x^2 + \frac{y^2}{4} = 1$
   
   (1) around the $x$ axis.
   
   (2) around the $y$ axis.

7. Find the volume $V$ of the solid generated by revolving about the $x$ axis the region between the graphs of $f(x) = \sin x$ and $g(x) = \cos x$ on the interval $[0, \frac{\pi}{4}]$. 