Math 141H Homework 11 (Section 9.1 – 9.3)  Due 4/16

Show all your work. Jumping to the right answer without minimum reasoning deserves no credit.

1. Find the Taylor polynomial $p_n$ of the following functions about 0 with given $n$.
   
   (1) $f(x) = \cos x, \quad n = 6$

   (2) $f(x) = e^{-x}, \quad n = 5$

   (3) $f(x) = \sin^{-1} x, \quad n = 2$

2. Evaluate the limit.
   
   (1) $\lim_{n \to \infty} \frac{1}{\tan^{-1} n}$

   (2) $\lim_{n \to \infty} n \sin \frac{\pi}{n}$

   (3) $\lim_{n \to \infty} \left(1 + \frac{1}{3n}\right)^n$

   (4) $\lim_{n \to \infty} \frac{3^n}{2^n}$

   (5) $\lim_{n \to \infty} \frac{\sqrt{n^2 - 1}}{n}$

3. Let $a_1 = 2$ and $a_{n+1} = 2 - \frac{1}{a_n}$.

   (1) Show that $\{a_n\}$ is bounded: $1 \leq a_n \leq 2$

   (2) Show that $\{a_n\}$ is decreasing. (Hint: Show that $a_{n+1} - a_n < 0$)

   (3) Find the limit.

4. Let $\{a_n\}$ be a sequence $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \ldots$, which in general $a_{n+1} = \sqrt{2 + a_n}$.

   (1) Show that $\{a_n\}$ is bounded: $a_n \leq 2$

   (2) Show that $\{a_n\}$ is increasing. (Hint: Show that $a_{n+1} - a_n > 0$)

   (3) Find the limit.