Math 141H Homework 2 (Section 6.4 and 6.5)

Show all your work. Jumping to the right answer without minimum reasoning deserves no credit.

1. Suppose 12 joules of work are required to extend a spring from its length of 1 meter to 3 meters. Find the work \( W \) done extending the spring from a length of 3 meters to 5 meters. (Hint: Calculate the constant \( k \) first.)

2. A conical tank with its point at the bottom has its top is 4 feet underground. The top of the tank has a radius of 5 feet and the tank is 10 feet tall. Suppose the tank is filled with water weighting 5 \( \text{lb/ft}^3 \) (instead of 62.5 \( \text{lb/ft}^3 \)).
   
   (1) Draw a picture of the situation.
   
   (2) Calculate the cross-sectional area.
   
   (3) Write down the integral for work \( W \) required to bring water to the ground until there is a depth of 5 foot of water in the tank.
   
   (4) Evaluate the integral (optional).

3. A tank is in the shape of the curve \( y = 8x^3 \) for \( 0 \leq x \leq 1 \) revolved around the \( y \) axis. Suppose the tank is filled with water weighting 5 \( \text{lb/ft}^3 \) (instead of 62.5 \( \text{lb/ft}^3 \)).
   
   (1) Draw a picture of the situation.
   
   (2) Calculate the cross-sectional area in terms of \( y \).
   
   (3) Write down the integral for work \( W \) required to bring water to the top of the tank until there is a depth of 1 foot of water in the tank with respect to the \( y \) integral.
   
   (4) Evaluate the integral (optional).

4. Find the center of mass \((\bar{x}, \bar{y})\) of the semicircular region consisting of the points \((x, y)\) such that \( x^2 + y^2 \leq R^2 \) and \( y \geq 0 \).

5. Consider the region \( R \) bounded by the graphs \( y = x^2 \) and \( y = 2x + 3 \).
   
   (1) Find the area of the region.
   
   (2) Calculate the moment \( M_y \) around the \( y \) axis.
   
   (3) Calculate the moment \( M_x \) around the \( x \) axis.
   
   (4) Find the center of mass \((\bar{x}, \bar{y})\).