Quizz and solution for Advanced Calculus, Math 410, section 0201 on 04/07/2014

The test lasts 10 minutes. No documents are allowed. The use of a calculator, cell phone or other equivalent electronic device is not allowed.

1) Give the $\varepsilon - \delta$ definition of the continuity of a function $f : S \rightarrow \mathbb{R}$ at a point $x_0 \in S$.

Solution. $\forall \varepsilon > 0$, $\exists \delta > 0$, $\forall x \in S$, if $|x - x_0|$ then $|f(x) - f(x_0)| < \varepsilon$.

2) At which points of $\mathbb{R}$ is the following function continuous

$$f(x) = 0 \quad \text{if } x \in \mathbb{N}, \quad f(x) = 1 \quad \text{if } x \notin \mathbb{N}?$$

Solution. The function is continuous at any $x \notin \mathbb{N}$. Indeed if $x \notin \mathbb{N}$ and $x_n \rightarrow x$ then there exists $N$ s.t. for any $n \geq N$, $x_n \notin \mathbb{N}$. Therefore for $n \geq N$, $f(x_n) = 1$ and converges to $f(x)$.

But $f$ is not continuous at $x$ if $x \in \mathbb{N}$. Because if $x \in \mathbb{N}$, we can find a sequence of approximations $x_n = x + 1/n$ s.t. no $x_n$ belongs to $\mathbb{N}$. Then $f(x_n)$ is constant equal to 0 and cannot converge to $f(x)$. 

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