1) Assume that a numerical method gives a sequence $x_n$ satisfying

$$x_n = x_0 \exp(-2^n).$$

What is the order of the method?

Solution. The method is of order 2: Indeed one has exactly

$$x_{n+1} = x_0 \exp(-2.2^n) = x_n^2 x_0^{-1},$$

so that

$$\frac{x_{n+1}}{x_n^2} \to x_0^{-1} \in (0, +\infty).$$

2) A sequence of approximation is defined by $x_{n+1} = \varphi(x_n)$. We assume that there exists $\bar{x}$ s.t. $\varphi(\bar{x}) = \bar{x}$ and $\varphi'(\bar{x}) = -0.5$. Does the sequence $x_n$ converge to $\bar{x}$ if $|x_0 - \bar{x}|$ is small enough? No proof is requested (just quote the result from class).

Solution. The sequence converges if $|x_0 - \bar{x}|$ is small enough. We saw in class that it was the case provided $|\varphi'(\bar{x})| < 1$ which is satisfied here.