CHARACTERIZATIONS OF MEASURES IN THE DUAL OF $BV$

MONICA TORRES
(JOINT WORK WITH NGUYEN CONG PHUC)

Abstract. For a bounded open set $\Omega$ with Lipschitz boundary, we characterize the measures in the dual space $BV_0(\Omega)^*$. We make precise the definition of $BV_0(\Omega)$, which is the space of functions of bounded variation with zero trace on the boundary of $\Omega$. This result extends, to bounded domains, a previous characterization of the signed measures in $\mathbb{R}^n$ that belong to $BV(\mathbb{R}^n)^*$ obtained by the authors and a characterization of the positive measures in $BV(\mathbb{R}^n)^*$ obtained by Meyers and Ziemer. We also discuss the space $BV^{n-1}(\mathbb{R}^n)$, defined as the space of all functions $u$ in $L^{n-1}(\mathbb{R}^n)$ such that $Du$ is a finite vector-valued measure. We show that $BV(\mathbb{R}^n)^*$ and $BV^{n-1}(\mathbb{R}^n)^*$ are isometrically isomorphic. As a consequence of our characterizations, an old issue raised by Meyers and Ziemer is resolved by constructing a locally integrable function $f$ such that $f$ belongs to $BV(\mathbb{R}^n)^*$ but $|f|$ does not. Moreover, we show that the measures in $BV^{n-1}(\mathbb{R}^n)^*$ coincide with the measures in $W^{1,1}(\mathbb{R}^n)^*$, the dual of the homogeneous Sobolev space $W^{1,1}(\mathbb{R}^n)$, and that the measures in $BV_0(\Omega)^*$ coincide with the measures in $W^{1,1}_0(\Omega)^*$. Finally, the class of finite measures in $BV(\Omega)^*$ is also characterized. The space $W^{1,1}(\mathbb{R}^n)^*$ is used in image processing to model the noise of an image. Moreover, the full characterization of the space $BV^*$ is still an open problem in geometric measure theory.

E-mail address: torres@math.purdue.edu
E-mail address: pcnguyen@math.lsu.edu

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