Midterm test for Advanced calculus, 410, on 10/5/2011

The test lasts 50 minutes. No documents are allowed. The use of a calculator, cell phone or other equivalent electronic device is not allowed.

1) (3pts) Prove that the following limit holds

\[ u_n = \frac{\exp(2 + (-1)^n)}{n} \to 0. \]

2) (3pts) Show that the following sequence is increasing

\[ u_0 = 1, \quad u_1 = 2, \quad \forall n \geq 0 \quad u_{n+2} = 2u_{n+1} - u_n. \]

Hint: Use induction and consider \( u_{n+1} - u_n \).

3) (3pts) Prove that no matter how we choose \( \alpha \in \mathbb{R} \), the following function is not continuous at 0

\[ f(x) = \sin(1/x) \quad \text{if } x \neq 0, \quad f(0) = \alpha. \]

4) (1pts) Let \((u_n) > 0\) be a given sequence with \( u_n \to 0 \). Build \( \sigma : \mathbb{N} \to \mathbb{N} \) strictly increasing s.t. for any other \( \sigma' : \mathbb{N} \to \mathbb{N} \) strictly increasing with \( \sigma'(k) \geq \sigma(k), \forall k \) one has

\[ u_{\sigma(n)} \geq u_{\sigma'(n)}, \quad \forall n. \]